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ABSTRACT

Part III of the study describes a model for completing the cost assessment (justification) process by accounting for the fixed joint costs: a "fair" and equitable mechanism is developed in the context of game-theoretic approach. An n-person game is constructed in which the "players" are the institutions served by the library, but whose characteristic function reflects the initial set-up costs of any group of institutions if a library were to be constructed to meet their requirements. A Shapley value solution to the game is used to derive a hierarchy of institutions which may be employed in allocating the fixed portion of the annual joint costs of operating the library system to determine the institutionally based fees. The model is applied to data from HAM-TMC Library and the proportion of fixed operating costs to be allocated to each institution is calculated. The main theoretical aspects of the game relevant to this study and selected references are appended. (Author/JPF)

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Research Report CCS 303

COSTS AND THEIR ASSESSMENT TO USERS OF A MEDICAL LIBRARY, PART III: ALLOCATING FIXED JOINT COSTS

by

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July 1977

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IR006 304

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PREFACE

This report covers the results of a study that began October 1, 1974 and ended March 31, 1977; originally scheduled for completion in two years, various problems extended the study period to 30 months. Viewed in terms of the goals established in the proposal, the study has succeeded. Indeed, to the extent that an operational cost assessment model is now being used as an adjunct to administrative decision making at the principal study site, the study can be judged to have had significant impact that will probably spread to other similar institutions.

The study team was co-directed by Abraham Charnes, Director of the Center for Cybernetic Studies (CCS) and University of Texas System Professor and Jesse H. Jones Professor of Biomathematics and Management Science, and Sam W. Hitt, former Executive Director of the Houston Academy of Medicine-Texas Medical Center (HAM-TMC) Library. Other members of the study team were:

1) Milton E. F. Schoeman (Senior Investigator), Associate Professor of Management at the Graduate School of Business, University of Texas at Austin.

2) Richard A. Lyders (Administrative Associate), current Executive Director of the HAM-TMC Library.

3) Diane Cole Eckels (Research Associate), Associate Director of the HAM-TMC Library.

4) Keith Russell (Project Coordinator), former Research Librarian at the HAM-TMC Library.

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6) John J. Rousseau (Research Assistant), Ph.D. candidate at the Graduate School of Business, University of Texas at Austin.

7) Donna Robinson (Clerical Assistant), former staff member at the HAM-TMC Library.

As can be seen from the composition of this team, the study was structured as a joint venture, using a team approach in pursuing the goals. The HAM-TMC Library was the "site" for information, data gathering, testing and implementing the results, while the University of Texas Center for Cybernetic Studies was the site for analytical and computational methods of development. Much of the detailed description of this arrangement and other information regarding the format and pursuit of this study can be found either in the formal proposal or the previously distributed "Progress Report: October 1, 1974-September 30, 1975."

This final report is divided into four parts, each describing a separate study issue:

Part I- "An Overview of Services, Users and Budgeting in a Medical Library" discusses the general administrative concerns prompting this study initially. Briefly stated these concerns can be summarized in the question "how can the cost of a library be equitably allocated to its supporters in justifying its budget?"

Part II- "Recovering Costs from Service Usage" describes a model for associating one portion of total library costs with usage; the model's output are a set of

charges that can be used to recover (justify) these usage-based costs.

Part III- "Allocating Fixed Joint Costs" describes a model for completing the cost assessment (justification) process by accounting for the fixed joint costs; a "fair" and equitable mechanism is developed in the context of a game-theoretic approach.

Part IV- "Differences in the Use of a Health Science Library by Various Users" is a summary analysis of sample and survey data collected to facilitate the structuring of the cost assessment models. The results correlate well with previously discussed observations and extend some of them into new directions.

Although some references are made to other parts in the report, each part is written as a reasonably self-contained unit. Thus, readers may selectively choose to focus on only those topics of immediate interest and not need to read the entire report.

It should be explicitly noted that the data and relations ascribed to the HAM-TMC Library are real. The output assessments provided in one part have actually been used in the most recent budgeting cycle. Further refinements in the model are necessary, but the current version is already providing the library administration with a unique and practical tool for developing and analyzing its budget. It is anticipated that similar successful application can be made in other libraries once information in this report is

circulated.

Finally, the assistance of many staff members and professionals, both at the University of Texas and the HAM-TMC Library, is gratefully, albeit anonymously, acknowledgeed. Without the support and wholehearted cooperation of the National Library of Medicine and its project monitoring staff this study could not have been undertaken and carried through. The study team accepts full responsibility for the contents of this report and any criticisms should be directed to them.

CCS, Austin; HAM-TMC Library, Houston

July, 1977

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PART III

ALLOCATING FIXED JOINT COSTS

III-1. Introduction

The provision of many public services typically exhibits certain characteristics which render economic analyses of their operations difficult to say the least. One such characteristic is decreasing average costs due to increasing returns somewhere; another is jointness of supply, that is, once a service is being provided to one user, it can as well be provided to another user, sometimes at even lower prices due to the decreasing costs. Thus, decreasing costs and jointness of supply create pecuniary external economies. If several users demand a service, the service can be jointly provided at a lower per unit cost to all than if it were provided to individuals. This is a way of saying that the marginal social costs of a user's demands differ from the marginal private costs (and in this case are lower). Lack of homogeneity of users also causes pecuniary external economies. For example, if there are two users, one with comparatively large demands, then the demands of the smaller user can be met at a considerably lower cost due to the decreasing costs. A further complicating factor is the existence of indivisibilities; i.e., the use of some factor in the provision of a service will depend on whether or not the service is offered at all, and will be constant over various ranges of level of service. Related to this is the problem of peak loads. The costs of many services depend more on peak than average loads since many of the resources are fixed and some provision is usually made to accommodate peaks of demand

rather than only average demand. Thus indivisibilities may also lead to decreasing costs, since once a service is being provided its level of operations may be increased up to some point at little or no increase in cost.

As discussed in an earlier part of this report, the pricing and cost allocating of library facilities and services provided for heterogeneous users entail all of the above difficulties. Already considered have been the costs of library operations that are relatively easily attributable to a particular direct service. In addition, economies of scale have been noted in some services. However, a large portion of the total operating budget is not easily attributable to any particular services; these are joint costs called the support costs which are essential for the library to exist. Part of these joint costs are related to input, such as books and journals purchased and catalogued. Some joint costs are relatively fixed regardless of the level of usage. For example, the technical services personnel and materials costs are dependent upon the number and types of publications purchased and policies of processing and are, therefore, related to the programs of the institutions using the library rather than actual usage of the collection. In contrast, and as previously considered, another portion of the joint costs varies with the amount of usage.

Thus, the total operating costs of a library might be described as having the following three components: direct costs for each service provided; variable joint costs related to usage of services; and fixed joint costs related to certain characteristics and functions of the institutions using the library.

III-2. Alternative Methods of Cost Recovery

Traditionally, libraries have been financially supported by a single parent organization such as an academic institution, local government agency, etc. For various reasons an alternative financial arrangement of generating revenues from the users of a library has been suggested. What types of charging procedures might then be appropriate?

Let us first distinguish two general categories of users: (1) institutional or departmental users and (2) individual users within the institutions or departments. Let us assume here that the institutions or departments are to be assessed to recover the total operating budget of the library. Henceforth the term user shall refer to the individual user and institution shall refer to the user institution or department.

Perhaps the simplest method of establishing fees for member institutions is on the basis of an equal fee for each institution. This type of assessment may be reasonable in some cases, but institutions may be understandably reluctant to pay fees which are unrelated to the way costs are incurred. For example, individual users from some institutions may use the library more frequently and use more expensive services and facilities; therefore, these users would be more costly to serve than users from other institutions. In addition, this approach treats all institutions as if they have identical goals and programs, thereby ignoring differences in potential information requirements among institutions. In short, this approach disregards both user and institutional differences in informational requirements.

An alternative method of assessment might be to base institutional fees on assumptions of institutional requirements of a library according to programs within institutions. Although this approach improves upon the first method by incorporating institutional differences it does not consider differences in the levels and types of usage between users from institutions with the same broad goals and programs. The method still ignores differences in individual user informational requirements and behavior.

A third method might be to base institutional fees strictly on the use of each service by users within each institution. This approach takes account of differences in user behavior but ignores the important factor of the need for the users from each institution to be able to access a certain type of library when required. A certain quality of library collection and certain types of services must be accessible for potential use depending upon the informational requirements of institutions. Such requirements for a potential accessibility of a library are relatively fixed in that professional librarians and books cannot be gathered at a moment's notice.

Clearly, institutional fees should be determined in a way that directly incorporates both individual user behavior and variation among institutions in informational requirements. Based on our previous description of total operating costs, viz. direct costs for each service offered, variable joint costs related to usage, and fixed joint costs related to certain characteristics and functions of the institutions using the library, a reasonable and equitable charging mechanism might recover the direct costs and variable portion of joint costs

through usage based fees as previously discussed, and the fixed joint costs through institutionally based (or "membership") fees. Thus, an institutional member of a library system would pay a fee which is partially based on the costs generated through its members' use of services and partially based on the costs of providing that type and quality of library that the institution would need to have available whenever its members wished to use it.

The precise nature of an optimal pricing scheme for public services which exhibit the characteristics outlined above has received much attention in the economic literature. Various proposals for pricing have included marginal-cost pricing, average-cost pricing, price discrimination, and multi-part tariffs. Objections have been raised to each of these based upon considerations of economic welfare, problems of financing, or difficulties of application. Throughout the various writings two general points of view may be distinguished, namely "fairness" and "functional efficiency" (1). In recent years economists have urged the latter. They have argued that if price is set above marginal cost, then some potential users will be prevented from obtaining services for which they would just be willing to pay the marginal cost. If there are economies of scale, the resulting deficit should be covered by government subsidy. This, of course, ignores the question of who must pay (in the form of government taxation) the required subsidy when marginal cost pricing is used in the presence of decreasing costs, i.e., it ignores income redistributional questions. If users do not pay total costs, then nonusers must pay in some way

(lump sum or other taxes). Thus, using only the Pareto criterion we cannot say whether the situation when marginal-cost pricing is used is superior to when it is not, since marginal-cost pricing under increasing returns makes users better off at the same time as nonusers are made worse off.

Coase (2,3) has argued that in addition to price covering the cost of the marginal unit it is necessary for efficiency that the costs of all units be covered by payments from users. If the price a user pays does not cover the total costs of his demands, then there are some factors used in supplying him which are not being accounted for in the price. The use of these factors will therefore not be economized on, and hence efficiency will not be obtained; thus, if someone does not wish to pay the total costs of supplying his demands, the value in his use of the factors used in supplying him is less than their value in some other use. The conclusion is that users' paying full costs of services received is both necessary for efficient allocation of resources and necessary in order to make definite welfare statements. The corollary is that it is also necessary that each user pay the costs of supplying his own demands.

We note here that if the primary objective in providing services is to redistribute income (so that some cross subsidization is desired as a matter of course) then the principle that each user pay the costs of supplying his demands would not apply. However, it should be applied whenever efficient use of resources is the primary concern.

A methodology for determining usage based fees has been presented in an earlier part of this report. We confine our attention in the

following sections to a method for determining institutionally based fees which incorporates notions from the mathematical theory of games.

III-3. A General Characteristic Function for the Cost Allocation Game.

The fixed portion of the annual joint costs of operating a library typically include (among other items) different proportions of the following:

costs associated with library acquisitions (such as purchases of books and serials, binding, etc.);

costs of different types of personnel;

certain administrative costs (such as legal and professional fees, supplies, etc.); and

expenditures on various types of equipment.

These costs are to be recovered through a set of institutionally based fees.

As we have already noted, these costs are not related to usage of services within the library. Rather, they are costs which must be incurred in order to maintain a certain quality and type of library for use by the institutions whenever their members might wish to take advantage of the facilities, regardless of how frequently (or infrequently) this might occur.

Moreover, although the library must accommodate the demands of all the member institutions, distinct differences in informational requirements and quality and type of library facilities demanded may exist among the member institutions. Such differences are determined by the various (non-usage related) characteristics peculiar to different institutions, for example, the set of academic programs offered, accreditation standards and potential user population.

Given these circumstances how might a set of fees (one per institution) be assessed that is efficient in that total annual fixed joint costs are recovered yet equitable in the sense that an institution's fee closely reflects that institution's contribution to costs?

A number of criteria of fairness and efficiency suggest themselves:

(i) each group of institutions should cover the costs of accommodating its demands, where the size of a group ranges from one to all institutions;

(ii) institutions with similar demands should be charged similar amounts;

(iii) institutions whose demands provide benefits for other institutions may be charged less.

In the limit these criteria are not necessarily fully compatible, so that one is looking for a scheme that reflects all of them so far as possible.

The notions of game theory enable us to provide an analytic formulation of such a "fairness plus efficiency" approach to cost allocation. The basic concepts are by now well documented, and various models, concepts of solution and applications to economic, social and political situations have been presented in the literature. We summarize in an appendix the main theoretical aspects relevant for our purpose.

The objective, therefore, is to construct an n-person game in which the "players" are the institutions served by the library (i.e., the member institutions of the library system) and whose characteristic

function reflects the fixed portion of annual joint costs generated by any institution or subset of institutions. In this way, any of the concepts of solution to this game can be employed as a basis for allocating these costs to member institutions.

A reasonable approximation to the overall level of the fixed portion of annual joint costs can be obtained (admittedly, in a somewhat round-about fashion) by subtracting estimates of usage related and certain other costs from the total annual operating budget. However, an accurate breakdown into the cost levels for individual components is exceptionally difficult to determine for two reasons. First, the various items that contribute to fixed joint costs are themselves difficult to isolate with any precision. Second, even if the list of contributors cited above were exhaustive (which it certainly is not), the required "proportions" (of costs of acquisitions, the different types of personnel, administration, etc.) are unknown and can only be guessed at.

Moreover, even if it were possible to derive a reasonably accurate estimate of the level of cost for each component of annual fixed joint costs, there is little or no evidence that systematically relates these cost levels back to the programs, accreditation standards, user population sizes, or other peculiarities of the institutions which generate these costs.

This near complete absence of data would appear to leave us with a hopeless task, but by making the following assumption we can partially resolve the difficulty and get a handle on the problem.

Recall that the relevant costs are those which must be incurred in order to maintain a certain quality and type of library as dictated by the member institutions' diverse informational requirements and demands for facilities, irrespective of library usage levels. Now the initial set-up costs of the library are the fixed joint costs of constructing and equipping this quality and type of facility in the first place, and are determined by this same set of (non-usage related) institutional characteristics. On this basis it would not appear totally unreasonable to assume (as a first approximation) that, for any group of institutions, the fixed portion of annual joint costs of operating a library to accommodate their joint requirements would be proportional to the fixed joint set-up costs of that library. Furthermore, it would also not be too far removed from reality to assume that the constant of proportionality would be the same for all groups of institutions, where the size of a group ranges from one to all institutions. In this case the relative ranking of institutions would remain the same for either initial set-up costs or fixed annual joint operating costs.

Since data on initial set-up costs are more easily obtainable, we can then construct a game in which the "players" are again the institutions served by the library but whose characteristic function now reflects the initial set-up costs that would be generated by any group of institutions if a library were to be constructed to meet that group's requirements. A solution to this game would then determine an allocation of joint set-up costs to member institutions, from which we can determine a hierarchy of the institutions based on their individual cost

allocations. Under the assumption that the relative ranking of institutions is the same for either initial set-up costs or fixed joint operating costs, we can then apply this hierarchy to the fixed portion of the annual joint costs of operating the library system to determine the institutionally based fees.

The remainder of this section will be devoted to the development of a general characteristic function for the game which reflects initial set-up costs for groups of institutions and to exploring, by means of one particular solution concept, how a hierarchy of institutions can be derived.

III-3.1 Development of a General Characteristic Function

What follows may be more clearly understood in the following context. Consider the situation in which a number of health related institutions are considering the construction of library facilities to provide for their various demands and requirements. It will be assumed that no institution has any existing facilities of its own or access to other facilities, and that joint ventures by different size groups are under consideration. The institutions might then be interested in the set-up costs of providing facilities to accommodate the different groups (including all of them together) as a first step in determining what will be built, for whom, and how much the users should pay.

Each institution may perform the functions of education, patient care and research, but to varying degrees and with different emphases. A program may be defined as a breakdown of any of these three functions in accordance with the particular objectives and priorities of an

institution. Thus we may distinguish between, for example, undergraduate and graduate teaching; training of nurses, doctors and dentists; child and adult patient care; various types of research, etc. Clearly, different institutions will have different requirements to support their various programs, with possible overlap among institutions both with regard to programs and requirements.

The initial costs which must be incurred for the provision of any library facility are here taken to be (i) purchases of books and journals, (ii) purchases of equipment and furnishings, and (iii) the construction cost of the building. The level of these costs will be determined by such factors as (a) the size of the potential user population, (b) the set of programs to be accommodated, and (c) accreditation standards for member institutions. The initial hiring costs of administrative, technical and public services personnel will not be included here in set-up costs, and, of course, the salaries of such personnel are part of annual operating costs and hence are also excluded here.

Let $N \equiv \{j:j=1,\ldots,n\}$ be the set of participating institutions and $M \equiv \{i:i=1,\ldots,m\}$ be the set of programs offered by all institutions together. Further, let M_j denote the set of programs conducted by institution j, $j \in N$, and $M_s \equiv \bigcup_{j \in s} M_j$ denote the set of programs conducted by any subset s of institutions. As already noted, we allow for two or more institutions to have programs in common, so that in general $M_j \cap M_k \neq \emptyset$, for $j, k \in N$. Finally let x_j denote the total potential membership (students and faculty) of institutions j, $j \in N$.

(i) Purchases of Books and Journals

We assume that a program must be supported by a specific minimum

core collection of books and journals if that program is to be conducted at all, irrespective of the number of people engaged in that program or of the institution to which they belong.

Let Y_i = set of books and journals required for a minimum core collection to support program i, $i \in M$.

We allow for different programs to have overlapping core collection requirements, so that in general $Y_i \cap Y_l \neq \emptyset$, i, $l \in M$.

Let $Y_s \equiv \bigcup_{i \in M_s} Y_i$ = set of books and journals required for a minimum core collection to support all programs conducted by subset s of institutions, $s \subseteq N$.

c_l = average unit cost of a book or journal. Then, letting $g_l(s)$ denote the cost to subset s of core collections to support all its programs, we can write

$$g_l(s) = c_l y_s , \qquad s \subseteq N \tag{1}$$

where $y_s \equiv |Y_s|$ = size (in number of books and journals) of Y_s.

(ii) Purchases of Equipment and Furniture.

The costs of certain types of furnishings such as carpets, drapes, and decorative material may be included as part of the costs of construction of the library building. The relevant items for our purposes here are book shelves, user study facilities (such as chairs, tables and lamps), and employee work facilities (such as desks, chairs, filing cabinets, etc.). All of these can meaningfully be related to potential user population, size of collections and personnel requirements (which, in turn, are determined by user population and collection size).

Let b_1 denote the average number of books and journals that can be accommodated per square foot of shelf space and c_2 denote the cost per square foot of shelf material. Let $g_2(s)$ denote the cost to subset s of providing shelf materials to accommodate its collections. Then we have that

$$g_2(s) = c_2 \cdot \frac{y_s}{b_1} , \qquad s \subseteq N \tag{2}$$

We assume that seating and study facilities must be available for a certain proportion, p_j, of the potential user population of institution j, $j \in N$. We may expect this proportion to vary according to the type of institution under consideration. For example, an institution that emphasizes clinical research or patient care will require seating for a smaller proportion of its membership than one heavily engaged in undergraduate or graduate teaching.

Let c_3 denote the average cost of study facilities (e.g., table, lamp, chair and/or lounge chair) and $g_3(s)$ denote the cost to subset s of providing these facilities for its membership. We then have that

$$g_3(s) = c_3 \sum_{j \in s} p_j x_j , \qquad s \subseteq N \tag{3}$$

The amount of staff work facilities (such as desks, chairs, filing cabinets, etc.) to be provided will depend upon the number of personnel required to operate the library. Library personnel may be categorized under three general headings according to the functions they perform: public services, technical services, and administration.

For any subset s of institutions let $Z^P(s)$, $Z^T(s)$, $Z^A(s)$ denote the desired number of pubiic, technical and administrative services personnel respectively.

Public services typically include answering reference questions, providing general information and help, reshelving material used in the library and check-out of material to be used outside the library. Personnel requirements for such services are then determined by the expected levels of these activities, in particular the number of circulations, which in turn can be related to either the size of the potential user population or that proportion of the population for which seating accommodation is to be provided. Following the latter we can write the desired number of public services personnel for any subset s of institutions as

$$z^P(s) = f^P\left(\sum_{j\in s} p_j x_j\right) , \qquad s \subseteq N \tag{4}$$

Technical services such as ordering, cataloging and binding are related to books; so that requirements for technical services personnel are more accurately defined as being a function of the size and type of collections rather than the number of users. Accordingly, we may write

$$z^T(s) = f^T(y_s) , \qquad s \subseteq N \tag{5}$$

The number of administrative personnel required is primarily determined by the number of other staff needed to operate the library rather than by collections or user population, so that we write

$$z^A(s) = f^A\left[z^P(s) , z^T(s) \right] , \quad s \subseteq N \tag{6}$$

i.e.,

$$z^A(s) = f^A\left[f^P\left(\sum_{j\in s} p_j x_j\right) , f^T(y_s) \right] , \quad s \subseteq N \tag{7}$$

Let c_4, c_5, c_6 denote the average costs of providing work facilities per public, technical and administrative services employee, respectively.

Then, for any subset s of institutions, the cost of providing work facilities may be denoted $g_4(s)$ and is given by

$$g_4(s) = c_4 Z^P(s) + c_5 Z^T(s) + c_6 Z^A(s), \qquad s \subseteq N \tag{8}$$

i.e.,

$$g_4(s) = c_4 f^P\left(\sum_{j\in s} p_j x_j\right) + c_5 f^T(y_s) + c_6 f^A\left[f^P\left(\sum_{j\in s} p_j x_j\right), f^T(y_s)\right] \qquad s \subseteq N \tag{9}$$

(iii) Building Construction Cost.

Essentially, we have three determinants of the size of library building required: space for collections of books and serials, study and browsing space for users, and work space for library personnel. Let b_2 denote the average number of books and journals that can be housed per square foot of library space (with due allowance for walkways between stacks of shelves). Then for subset s the square footage required to house its collections is given by

$$\frac{1}{b_2} \cdot y_s \quad , \qquad s \subseteq N \tag{10}$$

Let k denote the area (in square feet) required per study or seating facility. Then for subset s the area required for user study and seating facilities is given by

$$k \sum_{j\in s} p_j x_j \quad , \qquad s \subseteq N \tag{11}$$

Finally, let k^P, k^T, k^A denote the average number of square feet of work space required per public, technical, and administrative services employee, respectively. Then for subset s total employee work space is given by

$$k^P Z^P(s) + k^T Z^T(s) + k^A Z^A(s), \qquad s \subseteq N \tag{12}$$

The total area (in square feet) of a library to accommodate subset s of institutions may be denoted by K_s and is given by

$$K_s = \frac{1}{b_2} \cdot y_s + k \sum_{j \in s} p_j x_j + k^P Z^P(s) + k^T Z^T(s) + k^A Z^A(s), \quad s \subseteq N \tag{13}$$

In general the construction cost function will exhibit increasing returns to scale (decreasing average costs) over a broad range of capacities. Writing $g_5(s)$ as the construction cost for a library for subset s we then have that

$$g_5(s) = g(K_s), \qquad s \subseteq N \tag{14}$$

where $g' > 0$ and $g'' \leq 0$.

Let C(s) denote the initial set-up costs of building and equipping a library to meet the requirements of subset s of institutions. Combining equations (1), (2), (3), (8) and (14) we can then write

$$C(s) = g_1(s) + g_2(s) + g_3(s) + g_4(s) + g_5(s), \qquad s \subseteq N \tag{15}$$

That is, for $s \subseteq N$, (16)

$C(s) =$	$c_1 y_s$	cost of collections
	+	
	$c_2 \frac{y_s}{b_1}$	cost of shelf materials
	+	
	$c_3 \sum_{j \in s} p_j x_j$	cost of study facilities
	+	
	$c_4 Z^P(s) + c_5 Z^T(s) + c_6 Z^A(s)$	cost of employee work facilities
	+	
	$g(K_s)$	construction cost

where

$$Z^P(s) = f^P\left(\sum_{j\varepsilon s} p_j x_j\right)$$

$$Z^T(s) = f^T(y_s)$$

$$Z^A(s) = f^A\left[f^P\left(\sum_{j\varepsilon s} p_j x_j\right), f^T(y_s)\right]$$

$$K_s = \frac{1}{b_2} \cdot y_s + k\sum_{j\varepsilon s} p_j x_j + k^P Z^P(s) + k^T Z^T(s) + k^A Z^A(s)$$

We have derived a general expression for the initial costs of building and equipping a library to meet the requirements of any group of institutions. This cost function can then be used as the characteristic function for the "game" of allocating initial set-up costs to member institutions. By making some further assumptions about the functions f^P, f^T, f^A, Z^T, Z^A and g we are then in a position to calculate C(s) for any subset s of N and thus derive the characteristic function values for the game.

Before illustrating this procedure with a particular example, we pause to indicate how a solution to this game can be used to derive a hierarchy of institutions which may be employed in allocating the fixed portion of annual joint costs of operating a library.

III-3.2 Determination of a Hierarchy of Institutions

We have derived an n-person game (N,C), a solution to which allocates the initial set-up costs of constructing and equipping a library to the institutions served by the library.

Let $(c_1,\ldots,c_n)$ be a vector of optimal allocations (by whatever criteria) to the member institutions. We can rank these allocations

in decreasing order of magnitude to obtain (relabelling if necessary)

$$c_1 \geq c_2 \geq \ldots \geq c_n .$$

We may derive a set of indexes $I_1,\ldots,I_n$ in which the index for the j^{th} institution is the ratio of the cost allocation to the j^{th} institution to that of the largest cost allocation to all institutions. Thus,

$$I_j = \frac{c_j}{c_1} , \qquad j = 1,\ldots,n$$

and clearly $I_1 = 1.0$. We then have a hierarchy of institutions based upon the set of indexes in which

$$1 = I_1 \geq I_2 \geq \ldots \geq I_n > 0$$

The fixed portion of annual joint operating costs of the library is then allocated according to this hierarchy. Let F denote annual fixed joint operating costs. Then the allocation to the "largest" institution in the hierarchy is given by

$$X_1^* = \frac{F}{\sum_{j=1}^{n} I_j} \cdot I_1$$

and in general

$$X_j^* = \frac{F}{\sum_{k=1}^{n} I_k} \cdot I_j , \qquad j=1,\ldots,n.$$

III-4. Some Practical Simplifications and a Special Case

To make the model operational requires further assumptions about the various functions and parameters involved in the general expression (16) in order to calculate the cost to any subset of institutions of building and equipping a library to meet its requirements.

For the general n-person game there are 2^n-1 non-empty subsets to be considered, so that as the number of member institutions increases linearly the number of characteristic function values to be determined grows exponentially. Moreover, each calculation of a characteristic function value requires a series of lower level or "nested" computations involving functions of parameters and variables and functions of functions. Evidently, depending on which assumptions are made the calculations can soon become very cumbersome. In addition, even if all required computations were performed the solution procedure for even a moderate size game may still entail considerable time and effort.

The following simplifications should not appear unreasonable for many practical situations and can substantially reduce the amount of computation involved. Moreover, in specific cases the resultant form of the characteristic function can be shown to possess certain properties that further diminish the amount of work required and make for an easy solution to the game.

We first consider the equations defining personnel requirements, viz., equations (4), (5) and (7) of the previous section.

We assume constant ratios between the required number of public services personnel and planned seating and study capacity, and between the required number of technical services personnel and the size of collections. We further assume that the number of administrative personnel required is a constant fraction of the total number of other staff required to operate the library irrespective of their distribution over public and technical services. In this way we can rewrite

equations (4), (5), (7) and (13) as

$$Z^P(s) = f^P\left(\sum_{j\varepsilon s} p_j x_j\right), \qquad s \subseteq N \tag{17}$$

$$Z^T(s) = f^T y_s, \qquad s \subseteq N \tag{18}$$

$$Z^A(s) = f^A\left[f^P \sum_{j\varepsilon s} p_j x_j + f^T y_s\right], \qquad s \subseteq N \tag{19}$$

$$K_s = \left[\frac{1}{b_2} + k^T f^T + k^A f^A f^T\right] y_s + \left[k + k^P f^P + k^A f^A f^P\right] \sum_{j\varepsilon s} p_j x_j, \qquad s \subseteq N \tag{20}$$

where f^P, f^T and f^A are now redefined as follows:

f^P = required (constant) ratio of public services personnel to planned seating and study capacity,

f^T = required (constant) ratio of technical services personnel to size of collections,

f^A = required (constant) ratio of administrative services personnel to other staff.

Turning to the construction cost function, g, we can approximate it linearly by assuming a constant average cost, γ, per square foot of space. Equation (14) thus becomes

$$g_5(s) = \gamma K_s, \qquad s \subseteq N \tag{21}$$

Substituting equations (17) through (21) into equation (16) and collecting terms we obtain

$$C(s) = \alpha y_s + \beta \sum_{j\varepsilon s} p_j x_j, \qquad s \subseteq N \tag{22}$$

where

$$\alpha = \left[c_1 + \frac{c_2}{b_1} + c_5 f^T + c_6 f^A f^T + \frac{\gamma}{b_2} + \gamma k^T f^T + \gamma k^A f^A f^T\right] \tag{23}$$

$$\beta = \left[c_3 + c_4 f^P + c_6 f^A f^P + \gamma k + \gamma k^P f^P + \gamma k^A f^A f^P\right] \qquad (24)$$

Note that α and β are constants so that the characteristic function of the game is now reduced to a simple linear expression in two variables. That is, the cost to any group of institutions of building and equipping a library to meet that group's requirements is a linear function of (a) the size and type of collections of books and serials needed to support that group's programs and (b) the amount of seating and study capacity which that group needs to have available at all times. Moreover, these two variables reflect the essential (non-usage related) differences between institutions, namely, differences in the amount, quality and type of informational requirements and facilities demanded. The coefficients α and β reflect both the direct and indirect effects of these variables on cost.

There are several notions of solution to the game (N,C), in which the characteristic function is defined by equation (22), one of which is the Shapley value, Shapley (11). For the general n-person game (N,C) it can be shown that

$$\phi_j = \sum_{\substack{S \subseteq N \\ S \ni j}} \frac{(n-s)!(s-1)!}{n!} \left[C(S) - C(S-\{j\})\right] \qquad (25)$$

where ϕ_j = the Shapley value for player j, $j \in N$.

The summation ranges over all the subsets of N to which player j belongs, and $s \equiv |S|$ denotes the number of members of subset S.

In the present context the Shapley value calculates the increase in cost generated by each institution for each possible combination of institutions, and takes an average.

The rationale for the Shapley value is sometimes given in terms of the bargaining power which each player imagines he possesses. This power (as estimated by the player in question) is based on what his joining each coalition contributes to that coalition. The rationale may also be given via an axiomatization, Shapley (11).

The Shapley value solution is a unique vector for all games. In addition it has some very desirable properties. In perfectly symmetric games where each player is in exactly the same position vis-a-vis others, the Shapley value solution awards the same payoff to each player; in games where the positions of players differ, the Shapley value reflects this difference.

Besides determining a unique solution vector based solely on the characteristic function of the game, the Shapley value solution has built into it a certain equity principle. This solution might therefore be a strong contender for the status of a "normative" solution, i.e., one which "rational players" might want to accept.

In some situations the nature of the problem is such that the characteristic function exhibits a special structure that further diminishes the computation involved and leads to a simple procedure for the Shapley value solution.

We can consider the game (N,C), in which the characteristic function is defined by equation (22), to be a composite one comprised of the following two games:

$$(N,U) \quad \text{where } U(s) = \alpha y_s , \qquad s \subseteq N \qquad (26)$$

$$(N,V) \quad \text{where } V(s) = \beta \sum_{j \in s} p_j x_j , \qquad s \subseteq N \qquad (27)$$

The game (N,U) is that of allocating to member institutions all initial set-up costs associated either directly or indirectly with providing the size, quality and type of collections of books and serials necessary to support all programs. The game (N,V) is that of allocating all set-up costs incurred either directly or indirectly through the provision of seating and study facilities for library users.

Paraphrasing an axiom of Shapley (11): if the j^{th} institution is a "player" in both games, then the Shapley value solution for the cost allocation to that institution in the composite game is equal to the sum of the Shapley value solutions for the cost allocations to that institution in the separate games; that is,

$$\phi_j(C) \equiv \phi_j(U+V) = \phi_j(U) + \phi_j(V)$$

Now note that the game (N,V) is inessential, that is,

$$\sum_{j\in N} V(\{j\}) = \beta \sum_{j\in N} p_j x_j = V(N),$$

so that the core consists of the single point

$$(\beta p_1 x_1, \ldots, \beta p_n x_n).$$

This unique vector is also the Shapley value solution for the game (N,V), that is

$$\phi_j(V) = \beta p_j x_j \quad , \qquad j \in N \qquad (29)$$

Now consider the game (N,U). A general result due to Littlechild and Owen (8) is that a simple and easily calculated expression may be obtained for the Shapley value whenever the characteristic function is a "cost" function with the property that the cost of any subset of players is equal to the cost of the "largest" player in that subset. The cost function is then subadditive but its negative is superadditive.

The Shapley value of a game based on the negative of the cost function is minus the Shapley value of a game based on the cost function itself.

Following Littlechild and Owen we subdivide the set N of players according to type. Let N_i denote the set of players of type i, i=1,...m, and let $n_i>0$ denote the number of players of type i.

Thus

$$N = \bigcup_{i=1}^{m} N_i \qquad \text{and} \qquad n = \sum_{i=1}^{m} n_i \ .$$

Let u_i denote the "cost" associated with player type i. In the present context, therefore,

$$u_i = \alpha y_i \ , \qquad i = 1,\ldots,m \tag{30}$$

where y_i denotes the size of collections required to support all programs of institutions type i. We can order the players such that

$$0 = u_0 < u_1 < u_2 < \ldots < u_m \ .$$

Define the game on N by the subadditive cost function

$$U(\emptyset) = 0 \ , \quad U(S) = \max \{u_i\} \ , \tag{31}$$

with the maximization taken over all i such that

$$N_i \cap S \neq \emptyset \ .$$

Now define

$$R_k = \bigcup_{i=k}^{m} N_i \qquad \text{and} \qquad r_k = \sum_{i=k}^{m} n_i \ , \quad k=1,\ldots,m$$

Note that $R_1 = N$ and $r_1 = n$.

The Shapley value for the game (N,U) is given by

$$\phi_j(U) = \sum_{\substack{S \subseteq N \\ S \ni j}} \frac{(n-s)!(s-1)!}{n!} \left| U(S) - U(S-\{j\}) \right| \ , \quad j \in N$$

where $s \equiv |S|$

If the characteristic function is defined by (31), then the expression for the Shapley value may be simplified to (see Littlechild and Owen)

$$\phi_j(U) = \sum_{k=1}^{i} \frac{u_k - u_{k-1}}{r_k} \quad , \quad \text{for } j \epsilon N_i \ , \quad i=1,\ldots,m \tag{32}$$

Using equation (30) we can see that in the present context this becomes

$$\phi_j(U) = \alpha \sum_{k=1}^{i} \frac{y_k - y_{k-1}}{r_k} \ , \text{ for } j \epsilon N_i \ , \quad i=1,\ldots,m \tag{33}$$

Expression (33) can be interpreted as the following rule:

Divide the cost of catering for the smallest type of institution equally among all institutions. Divide the incremental cost of catering for the second smallest type of institution (above the cost of the smallest type) equally among all institutions except the smallest type, etc.

Two remarks are worth noting:

(a) If we set $\phi_j(U) = \Phi_i$ for $j \epsilon N_i$, then

$$\Phi_i = \Phi_{i-1} + (u_i - u_{i-1})/r_i$$

i.e.,

$$\Phi_i = \Phi_{i-1} + \alpha(y_i - y_{i-1})/r_i \tag{34}$$

(b) If $n_i = 1$ for all i, that is, there is only one player of each type, then

$$\Phi_i = \Phi_{i-1} + (u_i - u_{i-1})/(n-i+1)$$

i.e.,

$$\Phi_i = \Phi_{i-1} + \alpha(y_i - y_{i-1})/(n-i+1) \tag{35}$$

Thus, if the characteristic function U possesses the special property outlined above, the Shapley value of the original game (N,C) can be conveniently calculated as

$$\phi_j(C) = \phi_j(U) + \phi_j(V)$$

$$= \alpha \sum_{k=1}^{i} \frac{y_k - y_{k-1}}{r_k} + \beta p_j x_j \quad , \quad \text{for } j \in N_i \ , \quad i=1,\ldots,m \qquad (36)$$

Based on this Shapley value solution a hierarchy of institutions can be derived and used to allocate the fixed portion of annual joint operating costs as outlined in a previous section.

III-5. An Application: The HAM-TMC Library Experience

The Houston Academy of Medicine - Texas Medical Center (HAM-TMC) Library provides services and facilities for the following list of twenty-two institutions and colleges which can be categorized under six general headings.

Medical Schools:	Baylor University of Texas (UTMS)
Medical Associations:	Houston Academy of Medicine (HAM)
Nursing Schools:	Texas Women's University (TWU) U.T. System School of Nursing (UTSSN) Prairie View A&M University School of Nursing (PVAMUSN)
Graduate Medical and Health related Schools:	U.T. Graduate School of Biomedical Sciences (GSBS) U.T. School of Public Health (SPH) U.T. School of Allied Health (SAH) U.T. Dental Branch (DB)

Specialized Research Institutes:	U.T. System Cancer Center M.D. Anderson Hospital and Tumor Institute (UTMDA) Texas Institute of Rehabilitation and Research (TIRR) Texas Research Institute of Mental Sciences (TRIMS) Speech and Hearing Institute (SHI) Institute of Religion (IR)
Teaching and Research Hospitals:	Methodist Hospital St. Luke's Episcopal/Texas Children's Hospital (SL/TCH) Harris County Hospital District (HCHD) Hermann Hospital St. Joseph Hospital Memorial Hospital Veteran's Administration Hospital (VA)

The categories are necessarily broad with, in some cases, substantial variation in size and emphasis of institutions under a particular heading. In addition, the nature of the institutions is such that more than one classification is possible. For example, UTMDA might be classified as a graduate medical school, a research hospital, or a specialized research institute; SHI could also be considered as a graduate college; TIRR, TRIMS and IR are small in comparison with UTMDA and probably more closely resemble research hospitals than specialized research institutes; even the hospitals themselves vary as to their emphases on teaching and research.

In addition, Baylor Medical School includes smaller departments with programs in Allied Health and Biological Sciences, but data are not available on them in any disaggregated fashion, whereas they are available under the U.T. System. In the interests of comparability, therefore, we absorb SAH and GSBS into the U.T. Medical School, so that Baylor and U.T.M.S. may be considered to conduct essentially the same programs.

Medical schools require an in-depth collection on all fields of medicine as well as research material in specialized and peripheral areas. The requirements of medical associations tend to be as broad as their individual members' professional activities. Particularly at TMC the members of the Academy are often affiliated with one of the medical schools, hospitals, or research institutes, so that informational requirements can be expected to overlap with those of other institutions to a considerable extent.

Nursing schools require a basic collection in general medicine and nursing as do hospitals, although in the latter case the emphasis depends on the type of hospital considered.

Graduate medical and biomedical colleges require a smaller general medical collection than do the medical schools, but also use specialized research material beyond that which a medical school would normally require. Similarly with research institutes, but in the case of the smaller ones their specialized requirements overlap somewhat with those of large medical schools and graduate health related departments.

The HAM-TMC Library collection consists of approximately 133,000 books and serials, encompassing all fields of medicine, nursing and some special areas of research, different amounts of which are required by the different institutions to support their programs.

We do not distinguish here between the different types of literature that each institution would require; this would be extremely difficult to ascertain accurately in the present state of development of medicine and would necessitate the drawing of somewhat artificial boundaries

between program requirements. The general areas of education, research and patient care are widening rapidly in all types of medical institutions; existing fields are expanding and new fields are emerging in conjunction with developments in the physical sciences, and traditionally separate professional activities are becoming interrelated.

Instead, we proceed on the assumption that all institutions require a core collection consisting minimally of a certain amount of literature on general medicine and nursing. Differences in informational requirements between institutions are then reflected in the extent of this collection and the amount of other specific research items.

HAM-TMC Library personnel estimate, for example, that a teaching and research hospital requires a relatively small proportion of the HAM-TMC collection and that a nursing school requires all that a hospital does but additional material besides.

Small research institutes have requirements similar to those of research hospitals whereas large institutes tend to require substantially more; in both cases, depending upon the research emphasis, additional specific items may also be needed.

At the other end of the scale a medical school offering graduate programs in allied health and biomedical sciences and conducting research would require the bulk of the HAM-TMC collection in order to be considered a quality medical school. A quality medical school and nursing school together would require the entire collection if they were to build a library to accommodate their joint demands.

Two of the large graduate medical institutions, namely, the dental branch and school of public health, and the large research institute,

M.D. Anderson, require specialized material beyond that contained at the HAM-TMC Library. However, these institutions already have their own collections in the specialized materials relevant to their particular fields and essentially use the HAM-TMC collections as supplementary literature for back-up purposes only, although to varying degrees.

Prairie View A and M School of Nursing, TRIMS and VA hospitals also have their own libraries which contain much the same type of material as that at the HAM-TMC Library; but their collections are very much smaller and insufficient to fully support the programs they conduct.

Library personnel have estimated the number of books and serials in the HAM-TMC collection that each institution would require if it were to provide its own library in the absence of the HAM-TMC facilities; these figures are listed in Table III-1. Note that for those institutions with their own libraries (indicated by an asterisk) the relevant figure is the additional material to their current collection that would be required in the absence of the HAM-TMC Library.

Note further that in addition to combining the U.T. School of Allied Health and Graduate School of Biomedical Sciences with the U.T. Medical School (for comparability with Baylor), Table III-1 also includes the Speech and Hearing Institute and U.T. System School of Nursing with the U.T. Medical School. This grouping of institutions presents no real administrative problems since their fees traditionally come from the same source. If necessary, subdivision of the composite fee to be charged may be determined by a simple small game along the lines presented here.

TABLE III-1: INSTITUTIONAL REQUIREMENTS OF HAM-TMC COLLECTION

Institution	No. Books and Serials
Baylor	123,000
HAM	61,500
TWU	10,000
UTMS	133,000
GSBS	
SAH	
SHI	
UTSSN	
DB*	2,460
SPH*	6,150
UTMDA*	24,600
PVAMUSN*	6,150
TIRR	9,000
TRIMS*	6,750
IR	9,000
HCHD	9,000
Hermann	9,000
Methodist	9,000
SL/TCH	9,000
Memorial	9,000
St. Joseph	9,000
VA*	2,250

*own library

The advantage of this grouping is that the entire HAM-TMC collection is now required by this composite institution and every other institution requires only some proportion of this total in its core collection. In this way, since the figures in Table III-1 refer to the same general type of literature, we can now approximate the core collection requirements for any group of institutions by the requirements of the "largest" individual member of that group.

Although this one hundred percent overlap of institutional requirements may not be a strictly accurate description for a few of the coalitions, it certainly is the case for the overwhelming majority of the one-quarter million-plus involved, and it is to be expected that any error introduced will be relatively slight. Moreover, this approximation considerably simplifies calculation of the characteristic function values and the solution procedure, since the characteristic function is now of the special type discussed in the previous section.

Table III-2 contains the essential calculations for determining the Shapley value of this game which is then written as

$$\Phi_j(U) = \Phi_i = \alpha \sum_{k=1}^{i} \frac{y_k - y_{k-1}}{r_k} \quad , \quad \text{for } j \in N_i \ , \quad i=1,\ldots,10$$

It is difficult to be precise in determining the membership of each institution in the Texas Medical Center system since frequently faculty hold joint appointments at two or more establishments for various proportions of the time. However, based on returns submitted by the institutions over the last two years we have derived Table III-3.

Table III-3 lists estimates of the membership of each institution at the estimated capacity over the medium term, the fraction of member-

TABLE III-2: CALCULATIONS OF SHAPLEY VALUES

Type i	Institutions	No. of Institutions r_i	No. of Volumes y_i	$y_i - y_{i-1}$	r_i	$\frac{y_i - y_{i-1}}{r_i}$	$\sum_{k=1}^{i} \frac{y_k - y_{k-1}}{r_k}$
1	VA*	1	2,250	2,250	18	125.00	125.00
2	DB*	1	2,460	210	17	12.35	137.35
3	SPH* PVAMUSN*	2	6,150	3,690	16	230.63	367.98
4	TRIMS*	1	6,750	600	14	42.86	410.84
5	TIRR IR HCHD HERMANN METHODIST SL/TCH MEMORIAL ST. JOSEPH	8	9,000	2,250	13	173.08	583.92
6	TWU	1	10,000	1,000	5	200.00	783.92
7	UTMDA*	1	24,600	14,600	4	3,650.00	4,433.92
8	HAM	1	61,500	36,900	3	12,300.00	16,733.92
9	BAYLOR	1	123,000	61,500	2	30,750.00	47,483.92
10	UTMS (Plus SAH, GSBS, SHI, UTSSN)	1	133,000	10,000	1	10,000.00	57,483.92

*own library

TABLE III-3: MEMBERSHIP OF INSTITUTIONS

Institution	Medium-Term Average Membership	Proportion to be Accommodated	Medium-Term Average Membership to be Accommodated
j	X_j	P_j	P_jX_j
Baylor	3,030	1/4	758
HAM	2,000	1/100	20
TWU	950	1/4	238
UTMS (Plus SAH,GSBS, SHI, UTSSN)	3,450	1/4	863
DB*	750	1/8	94
SPH*	400	1/8	50
UTMDA*	450	1/8	56
PVAMUSN*	130	1/8	16
TIRR	25	1/5	5
TRIMS*	35	1/8	4
IR	65	1/10	7
HCHD	40	1/10	4
Hermann	20	1/10	2
Methodist	90	1/10	9
SL/TCH	80	1/10	8
Memorial	50	1/10	5
St. Joseph	35	1/10	3
VA*	36	1/20	2

*own library

ship to be provided with seating and study facilities, and the resulting number of users to be accommodated in the event each institution were to construct its own library.

In most cases average annual membership over the past two years is a good enough approximation to planned capacity. However, the U.T. Medical School is relatively new and still expanding and therefore its current membership is not representative of that planned for the medium-term future. Accordingly, we assume the Medical School has a planned-for capacity comparable to the current membership of Baylor, so that when the Speech and Hearing Institute and Nursing School are included we arrive at the combined figure in Table III-3.

The proportions, p_j, are those supplied by HAM-TMC personnel. Those institutions with their own libraries are assumed to have existing seating and study facilities equal to fifty percent of what they actually require, so that the figures in Table III-3 represent the additional facilities that would be required were they to go it alone. Consequently, the relevant proportions, p_j, are one-half of what they would be if these institutions had no other library.

The Shapley value solution to this (inessential) game is given by

$$\phi_j(V) = \beta p_j x_j \quad , \qquad j \in N \ .$$

The Shapley value solution to the composite game is then given by

$$\phi_j(C) = \alpha \sum_{k=1}^{i} \frac{y_k - y_{k-1}}{r_k} + \beta p_j x_j \quad , \quad j \in N_i \ , \quad i=1,\ldots,10$$

The solution to our game, and hence the hierarchy of institutions, can be obtained in a straightforward manner once values for the coefficients α and β have been determined. This hierarchy can then be

applied to the fixed portion of annual joint operating costs to derive the institutional base fees.

However, we can eliminate the problems which inevitably arise in obtaining reliable estimates for all the various factors that constitute α and β by the following procedure.

Since α is a constant, the hierarchy of institutions based solely on the allocation of those set-up costs associated either directly or indirectly with collections will be independent of α, and is completely determined by the data in the final column of Table III-2. This hierarchy is thus given by a set of weights $\{w_j^1\}$ in which the weight for any institution is given by the ratio of the Shapley value for that institution to that for the "largest" institution, i.e.,

$$w_j^1 = \frac{\phi_j(U)}{\max_k\{\phi_k(U)\}}$$

$$= \sum_{k=1}^{i} \frac{y_k - y_{k-1}}{r_k} \Big/ \sum_{k=1}^{10} \frac{y_k - y_{k-1}}{r_k} \quad , \quad j \epsilon N_i \ , \quad i=1,\ldots,10$$

which is independent of α.

If we can isolate that fraction, f, of annual fixed operating costs, T, associated with collections, we can then use this set of weights directly to obtain a first set of institutional allocations as follows:

$$b_j^1 = \frac{w_j^1}{\sum_k w_k^1} \cdot fT \ , \qquad j \epsilon N$$

where b_j^1 = allocation to institution j of annual fixed operating costs associated with collections.

Similarly, since β is a constant, the hierarchy of institutions based solely on the allocation of those set-up costs associated either directly or indirectly with providing seating and study facilities will be independent of β, and is completely determined by the data of Table III-3.

The corresponding set of weights $\{w_j^2\}$ in this second hierarchy is given by

$$w_j^2 = \frac{\phi_j(V)}{\max_k\{\phi_k(V)\}} = \frac{p_j x_j}{\max_k\{p_k x_k\}} , \qquad j\epsilon N$$

which is independent of β.

We can then apply this second set of weights directly to the remainder of annual fixed operating costs to obtain a second set of institutional allocations as follows:

$$b_j^2 = \frac{w_j^2}{\sum_k w_k^2} \cdot (1-f)T , \qquad j\epsilon N$$

where b_j^2 = allocation to institution j of annual fixed operating costs associated with seating and study facilities.

The sum of these two allocations b_j^1 and b_j^2 to institution j will then be that institution's contribution to annual fixed operating costs; that is, the base fee for institution j is given by

$$b_j = b_j^1 + b_j^2 , \qquad j\epsilon N .$$

We note here that in some cases it may not always be possible to have the weights as calculated above be complete descriptions of institutional differences. For example, two or more institutions of the same type with identical programs should require the same core

collection. However, they still might differ with respect to the extent to which they would gather this literature if equipping their own libraries. It is not inconceivable that some institutions would procure less than the stated amount while others would obtain more. Similarly, some institutions, if building their own libraries, might provide more extensive facilities than the minimum they actually require (e.g., more spacious, more ornate, etc.) while others might do the reverse.

Differences such as these may be reflections of the qualities of institutions. If such quality differences are presumed to exist and be important, then account may be taken of them by multiplying the original weights by appropriate factors either less than or greater than unity. Alternatively, one can attempt to incorporate quality differences directly in the calculation of the original weights. In either case the rest of the procedure is unchanged.

Approximately fifty-five percent of the fixed operating costs of the HAM-TMC Library for 1976 were estimated as attributable to annual acquisitions of books and serials and related activities such as binding and cataloging. Projected fixed operating costs for 1977 were $579,065. Using the figure of 55% as a typical yearly average then yields an amount of $318,486 as projected fixed operating costs associated with collections and the remaining $260,579 is thereby attributed directly and indirectly to the provision of study facilities. The appropriate weights and corresponding cost allocations are given in Table III-4.

TABLE III-4: INSTITUTIONAL WEIGHTS AND COST ALLOCATIONS[1]

Institution	Collections		Study Facilities		Base Fee
	Weight	Cost Allocation	Weight	Cost Allocation	
j	w_j^1	b_j^1 ($)	w_j^2	b_j^2 ($)	b_j ($)
Baylor	0.826038	93,500	0.878331	92,126	185,626
HAM	0.291106	32,950	0.023175	2,431	35,381
TWU	0.013637	1,542	0.275782	28,926	30,468
UTMS (Plus SAH, GSBS, SHI, UTSSN)	1.500000	169,788	1.000000	104,888	274,676
DB*	0.002389	270	0.108922	11,425	11,695
SPH*	0.006401	725	0.057937	6,077	6,802
UTMDA*	0.077133	8,731	0.064890	6,806	15,537
PVAMUSN*	0.006401	725	0.018540	1,945	2,670
TIRR	0.010158	1,150	0.005794	608	1,758
TRIMS*	0.007147	809	0.004635	486	1,295
IR	0.010158	1,150	0.008111	851	2,001
HCHD	0.010158	1,150	0.004635	486	1,636
Hermann	0.010158	1,150	0.002317	243	1,393
Methodist	0.010158	1,150	0.010429	1,093	2,243
SL/TCH	0.010158	1,150	0.009270	972	2,122
Memorial	0.010158	1,150	0.005794	608	1,758
St. Joseph	0.010158	1,150	0.003476	365	1,515
VA*	0.002175	246	0.002317	243	489
	2.813691	318,486	2.484355	260,579	579,065

[1] All figures rounded to nearest dollar.
*own library

Note that the U.T. Medical School weight for "collections" is 1.5 as opposed to 1.0. This is so because U.T. Medical School is a relatively new institution in the HAM-TMC Library system and consequently has not been charged in the past for the early years' acquisitions which other institutions, particularly Baylor, have gradually accumulated. Therefore, on equity grounds it was decided to "phase in" U.T. Medical School over five years, so that after that period its weight would be 1.0.

The figures in Table III-4 show that Baylor contributes almost one-third of the annual fixed operating costs, whereas U.T. Medical School plus other U.T. graduate institutions are allocated over one-half of the costs. As might be expected, the hospitals and small research institutes each contribute less than one-half of one percent of annual fixed operating costs. The base fees listed in this table are the same as those appearing as part of the total assessments shown in the previous part of this report in Table II-7.

As an aside we note that knowing the fraction f defined above we can then determine the base fees directly by deriving a composite "weight" or, rather, allocation factor for each institution in an obvious way.

$$b_j = b_j^1 + b_j^2 = \left[f \frac{w_j^1}{\sum_k w_k^1} + (1-f) \frac{w_j^2}{\sum_k w_k^2} \right] T$$

$$= \left[\frac{f w_j^1 \sum_k w_k^2 + (1-f) w_j^2 \sum_k w_k^1}{\sum_k w_k^1 \cdot \sum_k w_k^2} \right] T$$

so that

$$b_j = w_j T \qquad , \qquad j \in N$$

$$\text{where } w_j = \left[\frac{f w_j^1 \Sigma w_k^2 + (1-f) w_j^2 \Sigma w_k}{\Sigma w_k^1 \cdot \Sigma w_k^i} \right]$$

$$\text{and } \sum_j w_j = 1 \ .$$

Thus the composite weights $\{w_j\}$ are allocation factors that give directly the proportion of fixed operating costs to be allocated to each institution.

APPENDIX TO PART III

A game may be described as any situation or contest in which the participants have conflicting goals, from simple parlor games to complex economic and international situations. Game theory is an analysis of the logic underlying all such strategic conflict, under assumptions that each participant is "perfectly rational". Games may be classified in a variety of ways; for example, according to the number of participants, the degree of information that the participants have, or the extent to which they can cooperate with each other to their mutual benefit through the formation of coalitions.

Definition: For an n-person game, define $N \equiv \{1,\ldots,n\}$ to be the set of all players. Any non-empty subset of N (including N itself and all the one-element subsets) is called a coalition.

Definition: By the characteristic function of an n-person game is meant a real-valued function, V, defined on all the subsets of N, which associates with each subset $S \subseteq N$ a value from the set of real numbers. The value associated with subset S [written V(S)] is obtained, in principle from a zero-sum two-person game played between subset S and its complement N-S. This value is the minimax solution for this zero-sum game and represents what that subset of players could guarantee for itself (over the long run) irrespective of what the other players may do. It follows that $V(\emptyset) = 0$.

If S and T are disjoint subsets of N, then in many situations we have the case that they can accomplish at least as much by joining forces as by remaining separate. Hence the superadditivity property

$$V(S) + V(T) \leqslant V(S \cup T) \quad \text{for } S \cap T = \emptyset .$$

We note, however, that in some instances this superadditivity property does not accurately reflect the reality of the situation.

Definition: By an n-person game in characteristic function form is meant a real-valued function, V, defined on the subsets of N, satisfying $V(\emptyset) = 0$. The game is often written concisely as (N,V). The superadditivity property is frequently assumed also.

Any notion of solution to the game (N,V) should satisfy some reasonable criteria about the disbursement of payoffs among rational players. Moreover, the set of disbursements satisfying these criteria should be as small a fraction of the total set of possible disbursements as is possible.

Definition: An imputation for the n-person game (N,V) is an n-dimensional vector of payoffs (one to each player) $X = (X_1,\ldots,X_n)$ satisfying

(i) $X_i \geq V(\{i\})$

(ii) $\sum_{i\in N} X_i = V(N)$

The first condition is known as "individual rationality" and states that no player will accept a payoff less than that which he can obtain by himself. Condition (ii) is sometimes called "grand coalition rationality"; if all n players form a coalition and then divide the proceeds of the game among themselves, the sum of the individual payoffs to players must equal the payoff to the coalition as a whole.

The conditions defining an imputation appear to be desirable of any solution to a game (N,V). However, the set of all possible imputations is in general extremely large, too large to serve as a solution set. Therefore in order to determine which imputation will be obtained some

further restrictions need be applied to narrow down the set of imputations. Before considering this aspect we note one instance in which the problem is trivial: that is, when the set of imputations consists of a single element. In this case that unique imputation will be the obvious result. This gives rise to the distinction between essential and inessential games. It is clear (by superadditivity, repeated n times) that

$$V(N) > \sum_{i \in N} V(\{i\}) .$$

Now if equality holds in the above, the set of imputations will consist of only one point, namely $X_i = V(\{i\})$ for all $i \in N$.

Definition: A game (N,V) is essential if

$$V(N) > \sum_{i \in N} V(\{i\}) .$$

It is inessential otherwise.

Although it is the essential games which are generally of interest, this distinction will be of importance to us in the main paper.

To return to the question of selecting one imputation from the generally many available, several authors have attempted to narrow down the set of possible imputations by imposing further restrictions on the payoff vectors which either seem desirable per se or can be inferred from the particular circumstances of the game.

Definition: Let X and Y be two imputations, and let S be a coalition. Then X dominates Y through S (notation, $X \underset{S}{\succ} Y$) if

(i) $X_i > Y_i$ for all $i \in S \subseteq N$

(ii) $\sum_{i \in S} X_i < V(S)$

That is, if all members of S prefer X to Y ; and, in addition, coalition S is capable of getting what imputation X gives S.

Definition: Let X and Y be two imputations, then X dominates Y (notation, $X \succ Y$) if there is some coalition S such that X dominates Y through S.

A solution concept introduced by Gillies (4) is known as the core of a game.

Definition: The core of a game (N,V) is the set of all undominated imputations for the game.

We thus have the following theorem:

Theorem: The core of the game (N,V) is the set of all n-vectors X satisfying

(i) $\sum_{i \in N} X_i = V(N)$

(ii) $\sum_{i \in S} X_i \geq V(S)$ for all $S \subset N$

(See Owen [8] for details of a proof.)

The second restriction above is sometimes referred to as "group rationality", and is a direct extension of "individual rationality" to all proper subsets of the set of players. It states that no group of players (including the single member groups) will accept a combined payoff less than that group could achieve for itself by forming a coalition.

Recall that we have defined a fair and efficient cost allocation as one which recoups total costs, but such that the allocation to any group of institutions does not exceed the cost of accommodating that group's demands; that is, there is no cross-subsidization. If we denote

by f_i the cost allocation to institution i, and let C(S) denote the cost of accommodating the demands of subset S, then a fair and efficient cost allocation may be defined as the set of n-vectors $(f_1,\ldots,f_n)$ satisfying

$$(3) \qquad \sum_{i\in N} f_i = C(N)$$

$$(4) \qquad \sum_{i\in S} f_i \leq C(S) \qquad \text{for all } S \subset N.$$

Now note that a cost allocation may be considered as a negative "payoff". Accommodating this reverse of sign we can see that our fair and efficient set of cost allocations is precisely the core of a game (N,-C); where in this instance the "game" consists of allocating costs to the "players" who correspond to the various institutions whose demands generate these costs.

The core is a smaller set of payoff vectors than the set of imputations but has (at least) two important difficulties. First, the core may be empty, that is, in our context, a feasible cost allocation as we have defined it may not exist. In other words, "group rationality" may be too restrictive for many games. It is conceivable that in some circumstances certain coalitions might not be able to form: agreements may be difficult or prohibitively costly to make such that even if some players become dissatisfied with a particular outcome they may not be able to form new coalitions with other players; some group of players may get a combined payoff less than they could achieve on their own as a coalition, but may not be able to break up existing arrangements to form this coalition; some coalitions may be unstable with members being lured away to other groups. However, it can be shown that under

certain convexity conditions (which in our context imply increasing returns to scale or decreasing average costs) the core will be non-empty.

A second difficulty with the core is that if the core contains one point (i.e., if one feasible cost allocation exists) there will in general be many. The problem then is to select one out of the many candidates as being in some sense "optimal". A third difficulty is that even when the core exists it may not reasonably reward players whose participation in a coalition is essential to the actual acquisition of the coalition payoff.

Game theoretic solutions provide several alternative choices from this list of candidates, one of which is the Shapley value, Shapley (11). For the general n-person game it can be shown that

$$\phi_i = \sum_{\substack{S \subseteq N \\ S \ni i}} \frac{(s-1)!(n-s)!}{n!} \left[V(S) - V(S-\{i\})\right]$$

where ϕ_i = the Shapley value for player i, $i \in N$.

The summation ranges over all the subsets of N to which player i belongs, and $s = S$ denotes the number of members of subset S. The proof that the Shapley value solution $(\phi_1,\ldots,\phi_n)$ is an imputation is straightforward.

In the present context the Shapley value calculates the increase in cost generated by each institution for each possible combination of institutions, and takes an average. A rationale for the Shapley value is sometimes given in terms of the bargaining power which each player imagines he possesses. This power (as estimated by the player

in question) is based on what his joining each coalition contributes to that coalition. The rationale may also be given in another way, namely by means of an axiomatization. It can be demanded that any n-tuple of payoffs $(\phi_1,\ldots,\phi_n)$ for which a claim is made as a "reasonable" outcome of the n-person games, satisfy three axioms.

(i) the vector $(\phi_1,\ldots,\phi_n)$ should depend only on the characteristic function of the game, not on any inherent properties of the players, in particular not on the way the players are labeled;

(ii) it must be that $\sum_{i=1}^{n} \phi_i = V(N)$; and

(iii) $\phi_i(V+W) = \phi_i(V) + \phi_i(W)$

where V and W are the characteristic functions of two games. That is, if the i^{th} player is in both games then the minimal expected payoff accruing to him in the composite game shall be the sum of the minimal expected payoffs accruing to him in the separate games. It should be mentioned that Luce and Raiffa (7) have criticized this third axiom, but admit that they have no alternative concept of value to suggest. Nor do they appear to adequately value its pertinence for situations involving multiple games.

The most important and gratifying consequence of Shapley's three axioms is that they are sufficient to guarantee the existence of a unique payoff vector for all games which is additive over games. Moreover, this payoff vector is in the core if the game is "convex" and the core is nonempty. In addition it has other desirable properties. In perfectly symmetric games where each player is in exactly the same position vis-a-vis others, the Shapley value solution awards the same

payoff to each player; in games where the positions of the players differ, the Shapley value reflects this difference. Some advantages and disadvantages of the Shapley value concept have been summarized by Rapoport (9):

> "...the Shapley value solution, besides determining a unique disbursement of the payoffs solely by the characteristic function of the game, has built into it a certain equity principle. This solution might therefore be a strong contender for the status of a 'normative' solution, i.e., one which 'rational players' ought to accept. Its weakness is precisely in that it derives entirely from the characteristic function of the game and not from what is 'behind' the characteristic function, i.e., the strategic structure of the game itself rather than the bargaining positions of the players in the process of coalition formation."

Other solution concepts from game theory evolve from different considerations, such as the bargaining positions of the players in the process of coalition formation. Still others emphasize the degree of dissatisfaction experienced by the players with a particular outcome of the game (for example, with a particular cost allocation), and adopt as a solution that outcome which minimizes the maximum "complaint" by any participant or group of participants acting in unison, Schmeidler (10); or that outcome which minimizes the maximum tendency for any player or group of players to disrupt the solution, Littlechild and Vaidya (6).

The notions of game theory would thus appear to be a powerful tool in an analysis of the problem of determining an allocation of joint costs to the member institutions of a library complex (which exhibits the characteristics referred to earlier), which is acceptable to the institutions both as individual units and as groups, and which also satisfies certain criteria of fairness and efficiency.

REFERENCES

1. Bonbright, J.C., Principles of Public Utility Rates, Chapter 8, New York, 1961.

2. Coase, R. H., "The Marginal Cost Controversy," Economica, N.S., August 1946, 169-182.

3. Coase, R. H., "The Theory of Public Utility Pricing and Its Application," The Bell Journal of Economics and Management Science, 1, 1 (Spring 1970) 113-128.

4. Gillies, D. B., Some Theorems on N-Person Games, Ph.D. Thesis, Department of Mathematics, Princeton University, 1953.

5. Littlechild, S. C. and Owen, G., "A Simple Explanation for the Shapley Value in a Special Case," Management Science 20, 3 (November 1973) 370-372.

6. Littlechild, S. C. and Vaidya, K. G., "The Propensity to Disrupt and the Disruption Nucleolus of a Characteristic Function Game," International Journal of Game Theory 5, Issue 2/3 (1976) 151-161.

7. Luce, R. D. and Raiffa, H., Games and Decisions, New York, John Wiley and Sons, 1957.

8. Owen, G., Game Theory, W. B. Saunders Co., Philadelphia, 1968.

9. Rapoport, A., N-Person Game Theory, Ann Arbor: University of Michigan Press, 1969.

10. Schmeidler, D., "The Nucleolus of a Characteristic Function Game," The SIAM Journal of Applied Mathematics, 17, 6 (November 1969) 1163-1170.

11. Shapley, L. S., "A Value for N-Person Games," in Kuhn, H. W. and A. W. Tucker, Eds., Contributions to the Theory of Games, Volume 2, Princeton, New Jersey: Princeton University Press (1953).